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SINGLE-VENDOR SINGLE-BUYER OPTIMAL CONSIGNMENT POLICY FOR A SEASONAL PRODUCT

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ABSTRACT

In this paper, we consider single-vendor single-buyer consignment policy for a seasonal product. On profit maximization approach, recently literatures were presented on optimal consignment policy for the manufacturer under supply chain co-ordination and we include here the average inventory cost which is experienced for the sold products during the selling period. The inclusion of the selling period average inventory cost represents an improved model which is more realistic and significant in the real business world. The model is compared with previous one numerically and retail price markdown is analyzed for the managerial implications.

Key words: Consignment policy, newsboy problem, selling period average inventory, traditional system

INTRODUCTION

It has been shown from Sjoerdsma (1991) and Chen and Liu (2008) that in the consignment policy (CP) the manufacturer/vendor provides a per-unit commission on each sold item and a fixed incentive to the retailer/buyer so that the retailer can earn at least as much as in the traditional system (TS) while the retailer provides warehouse space for manufacturer to use for stock and pays the vendor only when products are sold. They both are motivated to sell more products and share information continuously. On the other hand, vendor and buyer independently play their roles in the traditional system (TS), i.e., the retailer buys the products from the vendor by paying wholesale price and keeps it in his/her warehouse and then sells it to the consumers at retail price. The retailers set their order quantity and the manufacturer determines the wholesale price to optimize their own profit.

The model by Chen and Liu (2008) is a milestone for the profit maximising approach and is developed on the basis of the newsboy problem. But the deficiency of the model is that the holding cost of those products which are sold gradually creating some inventories during the selling period is missed. This holding cost has a significant effect in the real business. Introducing this holding cost, we present a single-vendor single-buyer improved model for a seasonal product which is more realistic.

Unlike, most prior research focuses on models to minimise the cost of inventory in lieu of profit maximisation. Goyal (1977) proposed a joint economic lot size model which minimised the total cost of the supply chain. Williams (2000) showed that the inventory carrying costs of both the vendor and the retailer are reduced under a consignment model. Use of make-to-order production approach and internal information sharing with the vendor decreases the inventory holding cost as well as increases the supply chain profit (Kulp 2002). Notable research already has been done stating that the implication of CP is better than the implication of TS for both the vendor and buyer (Valentini and Zavanella 2003; Zhou and Wang 2007; Zavanella and Zanoni 2009; Hoque 2013). We have found, a lot of researchers have considered selling period average inventory of sold products in their studies (Sajadieh, Jokar and Modarres 2009; Yu-Jen Lin 2009; Hoque 2013). This literature review inspires us to do the present study which is more realistic and significant in business arena.

We organise the paper as follows: In section 2 we develop the model with uniform distribution of demand and find out its optimal solution. Section 3 presents a solution algorithm. Numerical illustrations and retail price markdown are shown in section 4 and section 5 respectively. Finally, section 6 concludes by highlighting the paper

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findings, managerial implications, limitations and future research directions.

2. Model

In this section, we first quickly review the model by Chen and Liu (2008). Consider a product facing stochastic demand to be uniform distribution x~u[μ – σ/2, μ + σ/2] with probability density function f(x). Its production cost c, retail price p, wholesale price w, holding cost (Operational part) h_r^{CP} carried by the retailer and (Financial part) h_m^{CP} carried by the manufacturer in CP and total holding cost h_r^{TS} carried by retailer in TS. The retailer can take decision only for order quantity Q to maximise its profit and let the retailer's shortage cost be s_r per unit for stock-out while the manufacturer incurs stock-out cost s_m for a product. On the other hand, the manufacturer offers per unit commission α on sold product to the retailer in CP and wants to maximise his profit based on the retailer's order quantity Q. Also, in the CP, the manufacturer insures the retailer to earn at least as much as in TS, by providing a fixed incentive A. In this regard, Chen and Liu consider π_r^{CP} and π_m^{CP} as the profit of the retailer and the manufacturer respectively in CP and defined as follows:

$$\pi_r^{CP} = \begin{cases} \alpha x - h_r^{CP}(Q-x) + A & ; \quad x \le Q \\ \alpha Q - s_r(x-Q) + A & ; \quad x > Q \end{cases}$$

$$\pi_m^{CP} = \begin{cases} (p-\alpha)x - cQ - h_m^{CP}(Q-x) - A & ; \quad x \le Q \\ (p-\alpha-c)Q - s_m(x-Q) - A & ; \quad x > Q \end{cases}$$

They find out an analytical solution of the model and they also have a traditional model in the similar approach to compare both models.

In usual business, products are sold gradually. If x number of products are sold in a period, then there will be an average inventory x/2 of the sold products because they are not sold at a time. We call it as selling period average inventory of sold products. Many researchers (e.g., Sajadieh, Jokar and Modarres 2009, Yu-Jen Lin 2009 and Hoque 2013) have considered selling period average inventory of sold products in their studies. Since, many factors of inventory holding cost (e.g., warehouse rent, insurance, tax etc.) depend on a fixed period rather than the actual holding period and the cycle time of a seasonal product for the retailer is very small, we consider the selling period average inventory of sold products is being kept over the full cycle time. Considering the above assumptions and notations, we improve the traditional and consignment policy models as follows:

2.1 Traditional system

The retailer's profit is given by

$$\pi_r^{TS} = \begin{cases} px - wQ - \tfrac{1}{2}xh_r^{TS} - h_r^{TS}(Q-x) & ; \quad x \le Q \\ (p-w)Q - \tfrac{1}{2}Qh_r^{TS} - s_r(x-Q) & ; \quad x > Q \end{cases} \qquad (1)$$

So, the retailer's expected profit is

$$E(\pi_r^{TS}) = \int_0^Q [px - wQ - h_r^{TS}(Q - \tfrac{1}{2}x)]f(x)dx + \int_Q^\infty [(p-w)Q - \tfrac{1}{2}Qh_r^{TS} - s_r(x-Q)]f(x)dx \qquad (2)$$

Since, demand x is uniformly distributed (i.e., x~u[μ – σ/2, μ + σ/2]) and E(π_r^{TS}) is concave in Q, use of Leibniz rule for differentiation under integration sign in dE(π_r^{TS})/dQ = 0 gives the optimal order quantity, Q^* of retailer as follows (as in appendix):

$$Q^* = (\mu - \frac{\sigma}{2}) + \left(\frac{p - w + s_r - \frac{1}{2}h_r^{TS}}{p + s_r + \frac{1}{2}h_r^{TS}} \right)\sigma \qquad (3)$$

The manufacturer's expected profit,

$$E(\pi_m^{TS}) = (w - c)Q^* \qquad (4)$$

Manufacturer's goal is to maximise his expected profit setting the wholesale price w, so, solution of dE(π_m^{TS})/dw = 0 gives the optimal wholesale price w^* as in equation (5).

$$w^* = \frac{\mu(p + s_r + \frac{1}{2}h_r^{TS})}{2\sigma} + \frac{p + 2c + s_r - \frac{3}{2}h_r^{TS}}{4} \qquad (5)$$

Evaluation of integral of equation (2) gives the optimal expected profit of the retailer as follows (see appendix):

$$E(\pi_r^{TS}) = \mu(p + \tfrac{1}{2}h_r^{TS}) - Q^*(w^* + h_r^{TS}) - \frac{1}{2\sigma}(p + s_r + \tfrac{1}{2}h_r^{TS})(Q^* - \mu - \frac{\sigma}{2})^2 \qquad (6)$$

2.2 Consignment policy

The profits of the retailer and the manufacturer are in equation (7) and (8) respectively.

$$\pi_r^{CP} = \begin{cases} \alpha x - \frac{1}{2} x h_r^{CP} - h_r^{CP}(Q-x) + A \ ; & x \le Q \\ \alpha Q - \frac{1}{2} Q h_r^{CP} - s_r(x-Q) + A \ ; & x > Q \end{cases} \quad (7)$$

$$\pi_m^{CP} = \begin{cases} (p-\alpha)x - cQ - \frac{1}{2} x h_m^{CP} - h_m^{CP}(Q-x) - A \ ; & x \le Q \\ (p-\alpha-c)Q - \frac{1}{2} Q h_m^{CP} - s_m(x-Q) - A \ ; & x > Q \end{cases} \quad (8)$$

So, the expected profits of the retailer and the manufacturer are given by equation (9) and (10) respectively.

$$E(\pi_r^{CP}) = \int_0^Q [\alpha x - h_r^{CP}(Q - \tfrac{1}{2}x)] f(x)dx + \int_Q^\infty [\alpha Q - \tfrac{1}{2} Q h_r^{CP} - s_r(x-Q)] f(x)dx + A \quad (9)$$

$$E(\pi_m^{CP}) = \int_0^Q [(p-\alpha)x - cQ - h_m^{CP}(Q - \tfrac{1}{2}x)] f(x)dx + \int_Q^\infty [(p-\alpha-c)Q - \tfrac{1}{2} Q h_m^{CP} - s_m(x-Q)] f(x)dx - A \quad (10)$$

Since, demand x is uniformly distributed (i.e., x~u[μ – σ/2, μ + σ/2]) and the expected profit functions are concave, use of Leibniz rule in dE(π_r^{CP})/dQ = 0 and dE(π_m^{CP})/dQ = 0 gives the optimal order quantity, Q^* of the retailer and for the manufacturer respectively by equation (11) and (12) (as in appendix):

$$Q^* = (\mu - \frac{\sigma}{2}) + \left(\frac{\alpha + s_r - \frac{1}{2} h_r^{CP}}{\alpha + s_r + \frac{1}{2} h_r^{CP}} \right) \sigma \quad (11)$$

$$Q^* = (\mu - \frac{\sigma}{2}) + \left(\frac{p - \alpha - c + s_m - \frac{1}{2} h_m^{CP}}{p - \alpha + s_m + \frac{1}{2} h_m^{CP}} \right) \sigma \quad (12)$$

The manufacturer offers per unit commission α for the sold products to the retailer such that the retailer optimal order quantity matches the manufacturer's one. So, equation (11) and (12) together sets the per unit commission as follows:

$$\alpha = \frac{\frac{1}{2} h_r^{CP}(2p - c + 2s_m) - s_r(c + h_m^{CP})}{c + h_r^{CP} + h_m^{CP}} \quad (13)$$

Evaluating the integrals of equation (9) and (10), we have the optimal expected profits of the retailer and the manufacturer in the algebraic forms respectively as below (see appendix):

$$E(\pi_r^{CP}) = \mu(\alpha + \tfrac{1}{2} h_r^{CP}) - Q^* h_r^{CP} - \frac{1}{2\sigma}(\alpha + s_r + \tfrac{1}{2} h_r^{CP})(Q^* - \mu - \frac{\sigma}{2})^2 + A \quad (14)$$

$$E(\pi_m^{CP}) = \mu(p - \alpha + \tfrac{1}{2} h_m^{CP}) - Q^*(c + h_m^{CP}) - \frac{1}{2\sigma}(p - \alpha + s_m + \tfrac{1}{2} h_m^{CP})(Q^* - \mu - \frac{\sigma}{2})^2 - A \quad (15)$$

To form a successful CP, the manufacturer ensures the retailer to earn at least as much as in TS (i.e., E(π_r^{CP})≥ E(π_r^{TS})). In this regard, the manufacturer provides the retailer the fixed fee A at least by equation (16).

$$A = E(\pi_r^{TS}) \ \mu(\alpha + \tfrac{1}{2} h_r^{CP}) + Q^* h_r^{CP} + \frac{1}{2\sigma}(\alpha + s_r + \tfrac{1}{2} h_r^{CP})(Q^* - \mu - \frac{\sigma}{2})^2 \quad (16)$$

Therefore, the total supply chain profit under CP is E(π_r^{CP})+E(π_m^{CP}).

The solution methodology we have discussed so far can be summarised and structured by the following algorithm.

3. Algorithm

Step 0: Initialize p, c, s_r, s_m, h_r^{TS}, h_r^{CP}, h_m^{CP}, μ and σ as given.

Step 1: Sequentially determine w^*, Q^*, E(π_r^{TS}) and E(π_m^{TS}) using equations (5), (3), (6) and (4) respectively.

Step 2: Determine α using equation (13) and again calculate Q^* using equation (11) or (12)

Step 3: Calculate sequentially A, E(π_r^{CP}) and E(π_m^{CP}) using equations (16), (14) and (15) respectively. Finally calculate the total supply chain profit by E(π_r^{CP})+E(π_m^{CP}).

4. Numerical illustration

Two numerical examples are taken from Chen and Liu (2008), where the market demand was considered with the uniform distribution over the domain [μ – σ/2, μ + σ/2] and the parameters p = \$30, c = \$10, $s_r = s_m = 0$, μ = 100 and $\sigma = 200$ are set as global. In the first one, holding cost is 36% of production cost (i.e., h_r^{TS} = 0.36c in TS and h_r^{CP} = 0.18c, h_m^{CP} = 0.18c in CP) and in the other one, holding cost is 20% of production cost (i.e., h_r^{TS} = 0.20c in TS and h_r^{CP} = 0.08c, h_m^{CP} = 0.12c are considered in CP). Various results are shown in Table 1 – Table 4 and by Graph 1. The expected profits of all parties in TS and CP calculated by previous and new models are presented in Table 1 and Table 2 respectively. The comparisons of the expected profits calculated by both models are in Table 3. Table 4 shows that the adaptation of CP not only increases the profit of the manufacturer by 50%, but also improves supply-chain profit by 33.33%. In Figure 1, the retailer's expected profit initially rises with the growth of demand uncertainty up to σ = 170 then falls. But the expected profits of the manufacturer and the supply-chain decrease gradually as the demand uncertainty increases.

Table 1. The expected profit of the supply chain in TS

	Results in Chen and Liu's model					Results in improved model				
h_r^{TS}	w^*	Q^*	E(π_r^{TS})	E(π_m^{TS})	Total profit	w^*	Q^*	E(π_r^{TS})	E(π_m^{TS})	Total profit
0.36c	20	59.52	297.62	595.24	892.86	19.10	57.23	260.41	520.82	781.23
0.20c	20	62.50	312.50	625.00	937.50	19.50	61.29	291.13	582.26	873.39

Table 2. The expected profit of the supply chain in CP

		Results in Chen and Liu's model						Results in improved model					
h_r^{CP}	h_m^{CP}	α	A	Q^*	E(π_r^{CP})	E(π_m^{CP})	Total profit	α	A	Q^*	E(π_r^{CP})	E(π_m^{CP})	Total profit
1.8	1.8	2.65	140.06	119.05	297.62	892.86	1190.48	3.31	122.55	114.47	260.41	781.23	1041.64
0.8	1.2	1.33	229.17	125.00	312.50	937.50	1250.00	1.67	213.50	122.58	291.13	873.39	1164.52

Table 3. Comparison of improved model and Chen-Liu model

Problems	Improved model decreases each expected profit in TS (Percentage)			Improved model decreases each expected profit in CP (Percentage)		
	E(π_r^{TS})	E(π_m^{TS})	Total profit	E(π_r^{CP})	E(π_m^{CP})	Total profit
1st Problem	12.50	12.50	12.50	12.50	12.50	12.50
2nd Problem	6.84	6.84	6.84	6.84	6.84	6.84

Table 4. The difference of expected profit in TS and CP calculated by improved model for first problem.

	Q^*	E(π_r)	E(π_m)	Total profit
TS	57.23	260.41	520.82	781.23
CP	114.47	260.41	781.23	1041.64
CP-TS	57.23	0	260.41	260.41
Increment	100%	0	50%	33.33%

Table 5. Price discount analysis on the expected profit under TS for the first problem

b	y(p)	μ	б	Price, bp	w	Q^*	E(π_r^{TS})	E(π_m^{TS})	Total profit
1.0	0	100	200	30	19.1	57.23	260.41	520.82	781.23
0.9	3	300	600	27	17.6	158.33	601.67	1203.33	1805.00
0.8	6	600	1200	24	16.1	283.72	865.35	1730.70	2596.05
0.7	9	900	1800	21	14.6	363.16	835.26	1670.53	2505.79
0.6	12	1200	2400	18	13.1	375.76	582.42	1164.85	1747.27
0.5	15	1500	3000	15	11.6	285.71	228.57	457.14	685.71

Table 6. Price discount analysis on the expected profit under CP for the first problem

b	y(p)	μ	б	Price, bp	α	A	Q^*	E(π_r^{CP})	E(π_m^{CP})	Total profit
1.0	0	100	200	30	3.31	122.55	114.47	260.41	781.23	1041.64
0.9	3	300	600	27	2.91	283.14	316.67	601.67	1805.00	2406.67
0.8	6	600	1200	24	2.51	407.22	567.44	865.35	2596.05	3461.40
0.7	9	900	1800	21	2.12	393.07	726.32	835.26	2505.79	3341.05
0.6	12	1200	2400	18	1.72	274.08	751.52	582.42	1747.27	2329.70
0.5	15	1500	3000	15	1.32	107.56	571.43	228.57	685.71	914.29

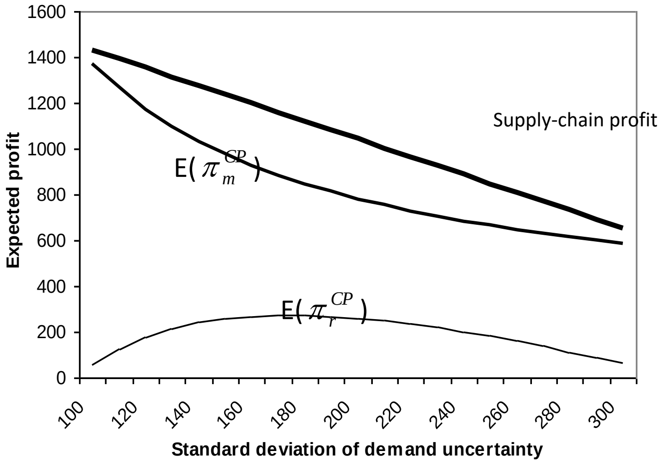

Figure 1. Expected profit in CP on demand uncertainty calculated by improved model for the first problem

CONCLUSIONS

This study shows that the addition of the cost of selling period average inventory of sold products represents a more realistic model for single-vendor single-buyer with a seasonal product. It proves that the implementation of CP can increase the manufacturer's profit as well as can improve the total supply-chain profit compared to TS. The numerical analysis in Table 3 indicates that compared to Chen and Liu, the profit of each party as well as the profit of entire supply chain is decreased by 6.84-12.5% due to implementation of the realistic holding cost. That means, the profit of each party calculated by Chen and Liu was erroneous by 6.48-12.5%. Retail price markdown notices to the manager that both parties will be more beneficial if price decreases to 24 from 30 for the first example.

This study has done on some assumptions and fixed demand distribution. It may be extended relaxing the assumptions and considering other type of demand for several vendor-buyers.

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APPENDIX

Proof of TS: The retailer's expected profit is as follows

$$E(\pi_r^{TS}) = \int_0^Q [px - wQ - h_r^{TS}(Q - \tfrac{1}{2}x)]f(x)dx + \int_Q^\infty [(p-w)Q - \tfrac{1}{2}Qh_r^{TS} - s_r(x-Q)]f(x)dx \quad \text{(A1)}$$

Taking derivative with respect to Q and setting it equal to zero, i.e., dE(π_r^{TS})/dQ = 0 implies

$$\frac{d}{dQ}[\int_0^Q [px - wQ - h_r^{TS}(Q - \tfrac{1}{2}x)]f(x)dx] +$$

$$\frac{d}{dQ}[\int_Q^\infty [(p-w)Q - \tfrac{1}{2}Qh_r^{TS} - s_r(x-Q)]f(x)dx]$$

$= 0$

Using Leibniz rule for differentiation under the integral sign, we have

$$-(w+h_r^{TS})\int_0^Q f(x)dx +$$

$(p - w - \frac{1}{2}h_r^{TS})$Q.f(Q)+

$$(p - w + s_r - \tfrac{1}{2}h_r^{TS})\int_Q^\infty f(x)dx$$

$-(p - w - \frac{1}{2}h_r^{TS})$Q.f(Q) $= 0$

Since, f(x) is uniform probability density function and demand x is uniformly distributed over the interval [μ – σ/2, μ + σ/2], we have

$$-(w+h_r^{TS})\int_0^Q f(x)dx +$$

$$(p - w + s_r - \tfrac{1}{2}h_r^{TS})(1 - \int_0^Q f(x)dx) = 0,$$

$$\int_0^Q f(x)dx = \frac{p - w + s_r - \frac{1}{2}h_r^{TS}}{p + s_r + \frac{1}{2}h_r^{TS}},$$

$$\int_{\mu-\frac{\sigma}{2}}^Q \frac{1}{\sigma}dx = \frac{p - w + s_r - \frac{1}{2}h_r^{TS}}{p + s_r + \frac{1}{2}h_r^{TS}},$$

$$Q = (\mu - \frac{\sigma}{2}) + \left(\frac{p - w + s_r - \frac{1}{2}h_r^{TS}}{p + s_r + \frac{1}{2}h_r^{TS}}\right)\sigma$$

Since, the expected profit function is concave in Q, the optimal order quantity of the retailer is $Q^* = (\mu - \frac{\sigma}{2}) + \left(\frac{p - w + s_r - \frac{1}{2}h_r^{TS}}{p + s_r + \frac{1}{2}h_r^{TS}}\right)\sigma$

The manufacturer's expected profit can be expressed as follows:

E(π_m^{TS}) = (w – c) Q^* = (w – c) ($\mu - \frac{\sigma}{2}$) + (w – c) $\left(\frac{p - w + s_r - \frac{1}{2}h_r^{TS}}{p + s_r + \frac{1}{2}h_r^{TS}}\right)\sigma$

Since, the above function is concave in w, dE(π_m^{TS})/dw = 0 gives the optimal wholesale price that is fixed by the manufacturer as w^* = $\frac{\mu(p + s_r + \frac{1}{2}h_r^{TS})}{2\sigma} + \frac{p + 2c + s_r - \frac{3}{2}h_r^{TS}}{4}$.

From (A1), we have

$$E(\pi_r^{TS}) = -Q(w+h_r^{TS})\int_0^Q f(x)dx + (p + \tfrac{1}{2}h_r^{TS})\int_0^Q x\,f(x)dx + Q(p - w + s_r - \tfrac{1}{2}h_r^{TS})\int_Q^\infty f(x)dx - s_r\int_Q^\infty x\,f(x)dx$$

$$= -Q(w+h_r^{TS})(1 - \int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}dx) + (p + \tfrac{1}{2}h_r^{TS})(\mu - \int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}x\,dx) + Q(p - w + s_r - \tfrac{1}{2}h_r^{TS})\int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}dx - s_r\int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}xdx$$

$$= -Q(w+h_r^{TS}) + \mu(p + \tfrac{1}{2}h_r^{TS}) + Q(p + s_r + \tfrac{1}{2}h_r^{TS})\frac{1}{\sigma}(\mu + \frac{\sigma}{2} - Q) - (p + s_r + \tfrac{1}{2}h_r^{TS})\frac{1}{2\sigma}((\mu + \frac{\sigma}{2})^2 - Q^2)$$

$$= \mu(p + \tfrac{1}{2}h_r^{TS}) - Q(w+h_r^{TS}) + \frac{1}{2\sigma}(p + s_r + \tfrac{1}{2}h_r^{TS})(Q - \mu - \frac{\sigma}{2})^2$$

Putting optimal values of Q and w, we have the optimal expected profit of the retailer as

$$E(\pi_r^{TS}) = \mu(p+\tfrac{1}{2}h_r^{TS}) - Q^*(w^* + h_r^{TS}) + \frac{1}{2\sigma}(p+s_r+\tfrac{1}{2}h_r^{TS})(Q^* - \mu - \frac{\sigma}{2})^2$$
(A2)

Proof of CP: The expected profits of the retailer and the manufacturer are as follows

$$E(\pi_r^{CP}) = \int_0^Q [\alpha x - h_r^{CP}(Q - \tfrac{1}{2}x)]f(x)dx + \int_Q^\infty [\alpha Q - \tfrac{1}{2}Qh_r^{CP} - s_r(x-Q)]f(x)dx + A$$
(A3)

$$E(\pi_m^{CP}) = \int_0^Q [(p-\alpha)x - cQ - h_m^{CP}(Q-\tfrac{1}{2}x)]f(x)dx + \int_Q^\infty [(p-\alpha-c)Q - \tfrac{1}{2}Qh_m^{CP} - s_m(x-Q)]f(x)dx - A$$
(A4)

From equation (A3), $dE(\pi_r^{CP})/dQ = 0$ implies

$$-h_r^{CP}\int_0^Q f(x)dx + (\alpha - \tfrac{1}{2}h_r^{CP})Q.f(Q) + (\alpha + s_r - \tfrac{1}{2}h_r^{CP})\int_Q^\infty f(x)dx - (\alpha - \tfrac{1}{2}h_r^{CP})Q.f(Q) = 0$$

Since, f(x) is uniform probability density function and demand x is uniformly distributed over the interval [μ – σ/2, μ + σ/2], we have

$$-h_r^{CP}\int_{\mu-\frac{\sigma}{2}}^Q f(x)dx + (\alpha + s_r - \tfrac{1}{2}h_r^{CP})(1 - \int_{\mu-\frac{\sigma}{2}}^Q f(x)dx) = 0,$$

$$\int_{\mu-\frac{\sigma}{2}}^Q \frac{1}{\sigma}dx = \frac{\alpha + s_r - \frac{1}{2}h_r^{CP}}{\alpha + s_r + \frac{1}{2}h_r^{CP}},$$

As function (A3) is concave in Q, so the optimal order quantity of the retailer is

$$Q^* = (\mu - \frac{\sigma}{2}) + \left(\frac{\alpha + s_r - \frac{1}{2}h_r^{CP}}{\alpha + s_r + \frac{1}{2}h_r^{CP}}\right)\sigma$$
(A5)

Similarly, from equation (A4), we have the optimal order quantity for the manufacturer is as follows:

$$Q^* = (\mu - \frac{\sigma}{2}) + \left(\frac{p - \alpha - c + s_m - \frac{1}{2}h_m^{CP}}{p - \alpha + s_m + \frac{1}{2}h_m^{CP}}\right)\sigma$$
(A6)

The manufacturer wants to face so much order from the retailer that also optimize his/her own expected profit. This is why, he/she offers per unit commission α for each sold product to the retailer such that order quantities of both party become same. So, equations (A5) and (A6) give,

$$\frac{\alpha + s_r - \frac{1}{2}h_r^{CP}}{\alpha + s_r + \frac{1}{2}h_r^{CP}} = \frac{p - \alpha - c + s_m - \frac{1}{2}h_m^{CP}}{p - \alpha + s_m + \frac{1}{2}h_m^{CP}},$$

$$\alpha = \frac{\frac{1}{2}h_r^{CP}(2p - c + 2s_m) - s_r(c + h_m^{CP})}{c + h_r^{CP} + h_m^{CP}}$$

From (A3), we have the algebraic form of the expected profit of the retailer as given below:

$$E(\pi_r^{CP}) = -Qh_r^{CP}(1 - \int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}dx) + (\alpha + \tfrac{1}{2}h_r^{CP})(\mu - \int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}x\,dx) + Q(\alpha + s_r - \tfrac{1}{2}h_r^{CP})\int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}dx - s_r\int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}xdx + A$$

$$= -Qh_r^{CP} + \mu(\alpha + \tfrac{1}{2}h_r^{CP}) + Q(\alpha + s_r + \tfrac{1}{2}h_r^{CP})\int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}dx - (\alpha + s_r + \tfrac{1}{2}h_r^{CP})\int_Q^{\mu+\frac{\sigma}{2}} \frac{1}{\sigma}xdx + A$$

$$= -Qh_r^{CP} + \mu(\alpha + \tfrac{1}{2}h_r^{CP}) + \frac{Q}{\sigma}(\alpha + s_r + \tfrac{1}{2}h_r^{CP})(\mu + \frac{\sigma}{2} - Q)$$

$$- \frac{1}{2\sigma}(\alpha + s_r + \tfrac{1}{2}h_r^{CP})((\mu + \frac{\sigma}{2})^2 - Q^2) + A$$

$$= \mu(\alpha + \tfrac{1}{2}h_r^{CP}) - Qh_r^{CP} - \frac{1}{2\sigma}(\alpha + s_r + \tfrac{1}{2}h_r^{CP})(Q - \mu - \frac{\sigma}{2})^2 + A$$

So, the optimal expected profit of the retailer is found by using optimal value of Q as follows:

$$E(\pi_r^{CP}) = \mu(\alpha + \tfrac{1}{2}h_r^{CP}) - Q^* h_r^{CP} - \frac{1}{2\sigma}(\alpha + s_r + \tfrac{1}{2}h_r^{CP})(Q^* - \mu - \frac{\sigma}{2})^2 + A$$

Using similar approach in equation (A4), we get the optimal expected profit of the manufacturer as given below:

$$E(\pi_m^{CP}) = \mu(p - \alpha + \tfrac{1}{2}h_m^{CP}) - Q^*(c + h_m^{CP}) - \frac{1}{2\sigma}(p - \alpha + s_m + \tfrac{1}{2}h_m^{CP})(Q^* - \mu - \frac{\sigma}{2})^2 - A$$

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